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A COMPARISON OF HEURISTIC METHODS USED IN HIERARCHICAL PRODUCT--ETC (U)
MAR 79 E A HAAS, A C HAX, R E WELSCH

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A COMPARISON OF HEURISTIC METHODS
USED IN HIERARCHICAL PRODUCTION PLANNING

by

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FOREWORD

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ABSTRACT

Hierarchical planning systems support medium range planning decisions in a batch processing production environment. In this approach, higher level (tactical) decisions impose constraints on lower level (operational) actions. Several heuristic approaches to hierarchical production planning have been proposed in the management science literature. ~~The purpose of this paper is to compare,~~ conceptually and empirically, four of these approaches.

The paper begins by discussing the direct optimization approach, and its associated drawbacks. The second section of the paper briefly describes several approaches to the design of a hierarchical production planning system and the distinguishing characteristics of the resulting algorithms. The third section of the paper compares four different methodologies for disaggregating tactical plans in a hierarchical setting. The paper concludes with recommendations for specific approaches to disaggregation in differing production environments.

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INTRODUCTION

The problem of planning and scheduling the production of a large number of items over time involves a multitude of decisions: manpower planning, use of overtime, buildup of seasonal inventory, frequency of setting up for the production of specific goods and associated run lengths. These problems have been approached from two distinct perspectives - attempts have been made to build an all inclusive direct optimization model, and heuristic algorithms have been proposed to take advantage of the hierarchical structure of production planning decisions (e.g. Holt, Modigliani, Muth and Simon [12], Winters [18], Hax and Meal [11], and Bitran and Hax [1]).

The essence of the direct optimization approach is to capture the indivisibilities of run lengths and economies of scale. This necessitates planning at a level of detail where setup costs are incurred. Unfortunately, the resulting mixed-integer programming formulation is too large to be solved by existing computational algorithms. Thus there has been major emphasis on transforming the problem into an equivalent linear programming model which can be solved by using large scale programming methods. Pioneering work in this area was done by Manne in 1958 [14]. This work was continued by Dzielinski, Baker and Manne [5], Dzielinski and Gomory [6], and later by Lasdon and Terjung [13]. Theoretically, these approaches guarantee an optimal solution and are a great contribution to the operations management literature. However, there are two principal drawbacks to the implementation of direct optimization models - long range planning is not directly supported, and the models do not facilitate managerial interaction with the solution process.

The data required by the direct optimization approach are inappropriate for supporting aggregate production decisions. Tactical decisions are based on long run production plans for aggregate products. However, direct optimization is forced to deal with products at the most detailed level, in order to include all relevant costs, so to support tactical planning, detailed product data must be aggregated. For example, a critical input for direct optimization is forecasted demand for each item over the entire planning horizon, typically one year. The use of detailed data in these situations has two major drawbacks.

- 1) Detailed forecasting requires significantly more computation than aggregate forecasting, thus not allowing as sophisticated modeling techniques to be used.
- 2) Forecasting at the item level and then aggregating the forecasts results in greater errors than aggregate forecasting.

A second practical disadvantage of direct optimization is the inability to coordinate managerial interaction with the model's solution. Managerial interaction in terms of tactical and operational decisions is desirable. For example, tactical decisions may involve manpower planning and seasonal inventory buildup. Management may feel that there are special reasons to limit overtime, other than available manpower. If they plan on adding a third shift in the near future, management may feel that it would be bad for workers' morale to become accustomed to overtime hours which would be unavailable after the addition of the extra shift. Direct optimization prohibits the coordination of these kinds of decisions with the model's output, as the model completely specifies the production plan. If top management wants to limit overtime, it is difficult to determine exactly how to alter the planned production of the thousands of specific products in

order to meet its goal. Moreover, it is virtually impossible to incorporate the goals of several levels of management into a single model. In practice, corporations that use the direct optimization approach find that changes made at differing managerial levels are frequently in conflict with tactical and operational planning decisions.

The authors favor hierarchical production planning over direct optimization. The strengths of hierarchical production planning will be emphasized as we describe this approach.

The objective of this paper is to compare four different heuristic methods for disaggregation within a hierarchical production planning framework: the Equalization-of-Run-Out-Time approach [12], the Winters approach [18], the Hax-Meal approach [11], and the Knapsack approach [1]. A brief summary of the structure of the hierarchical model and how these four methods differ is presented in the second section of this paper. The third section of the paper describes the statistical comparisons we used on simulated data. A section summarizing our results concludes the paper.

THE HIERARCHICAL APPROACH

The hierarchical approach is characterized by its recognition of the need to separate tactical from operational decisions and by its ability to deal with individual decisions at each level while using linking mechanisms for transferring higher level results to lower levels. Hierarchical production planning is designed to encourage managerial interactions at all levels (see Figure 1). Lack of managerial interaction is a weakness of direct optimization not present in hierarchical production planning.

The first level of the hierarchy determines an aggregate production plan: the timing of inventory buildups, manpower decisions, and capacity and overtime schedules for the entire planning horizon, typically a full year.

Given an aggregate production schedule for the entire planning horizon, a variety of methods have been proposed for disaggregating the production schedule for the current time period. Basically, the second level of the model attempts to minimize setup costs subject to constraints imposed by the aggregate production plan.

The third and final level in the hierarchy schedules the production quantities of each item in order to maximize the time until another setup is required and maintains the constraints imposed by the previous levels. Figure 1 shows the overall conceptualization of the hierarchical planning effort.

For this planning structure three levels of product aggregation have been identified - types, families, and items [11]. Items are defined as the specific products and are used only at the most detailed level of planning. Items sharing major setup costs are grouped into a family. This is done in order to allow all items in a family to be produced jointly, thereby avoiding unnecessary setup costs. At the highest level of aggregation, families with similar costs and productivity characteristics are grouped into product types. This grouping is used to determine the optimum aggregate levels of production, manpower, and inventory.

The Aggregate Model

The first level in hierarchical production planning is the aggregate level. The model used in our simulation was a linear program designed to minimize the costs involved in accumulating seasonal stock, regular and overtime production costs, and holding costs. Other costs can easily be added to the formulation. Any aggregate production planning model can be used in the first stage of hierarchical production planning as long as it adequately represents the practical problem under consideration. For extensive discussions of possible models, see [4], [8].

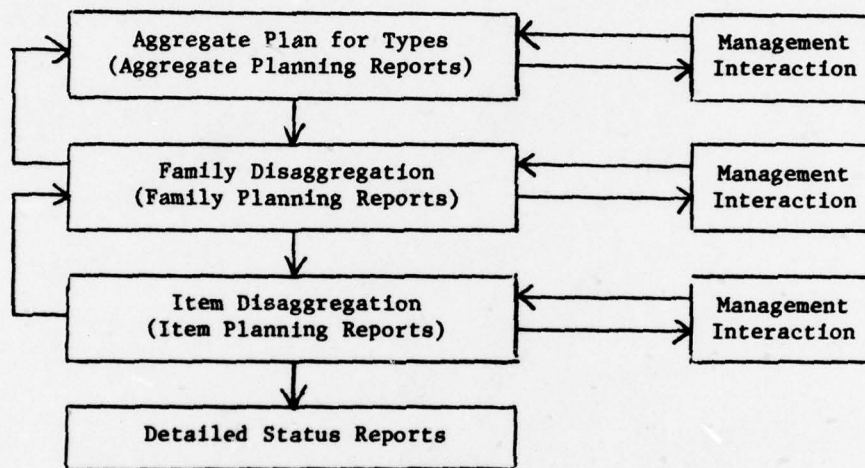


Figure 1: Conceptual Overview of Hierarchical Planning System

The model finds the optimal feasible solution for an entire year. Tactical managerial decisions may affect the solution of the model at this stage. This aggregate model is run every period¹ with a rolling horizon. All four of the disaggregation methodologies tested use as a primary input the aggregate production levels specified by the aggregate model.

The annual forecasts required for the model are of aggregate product types. Therefore, the forecast accuracy associated with the hierarchical approach is greater than that associated with direct optimization, which requires annual forecasts for each item. Since decisions on regular time, overtime, hiring and firing, and other production-rate parameters are based on total demand, more accurate forecasts of total demand should improve the model's decision-making ability.

The Disaggregation Methodologies

A critical step in the hierarchical scheme is to determine how aggregate production quantities should be allocated among the families belonging to each product type. It is at this level in the hierarchy where setup costs are considered. To insure feasibility and consistency in the system, the sum of the production of the families in each product type must not be greater than, and is ideally equal to, the amount dictated by the aggregate model for this product type. This paper compares four distinct disaggregation methods proposed in the literature: Equalization-of-Run-Out-Times (EROT) [12], Winters [18], Hax and Meal [11], and Knapsack [1]. Figure 2 displays a flow chart comparing the algorithms.

¹ A period is defined as 4 weeks, and a year as 13 periods.

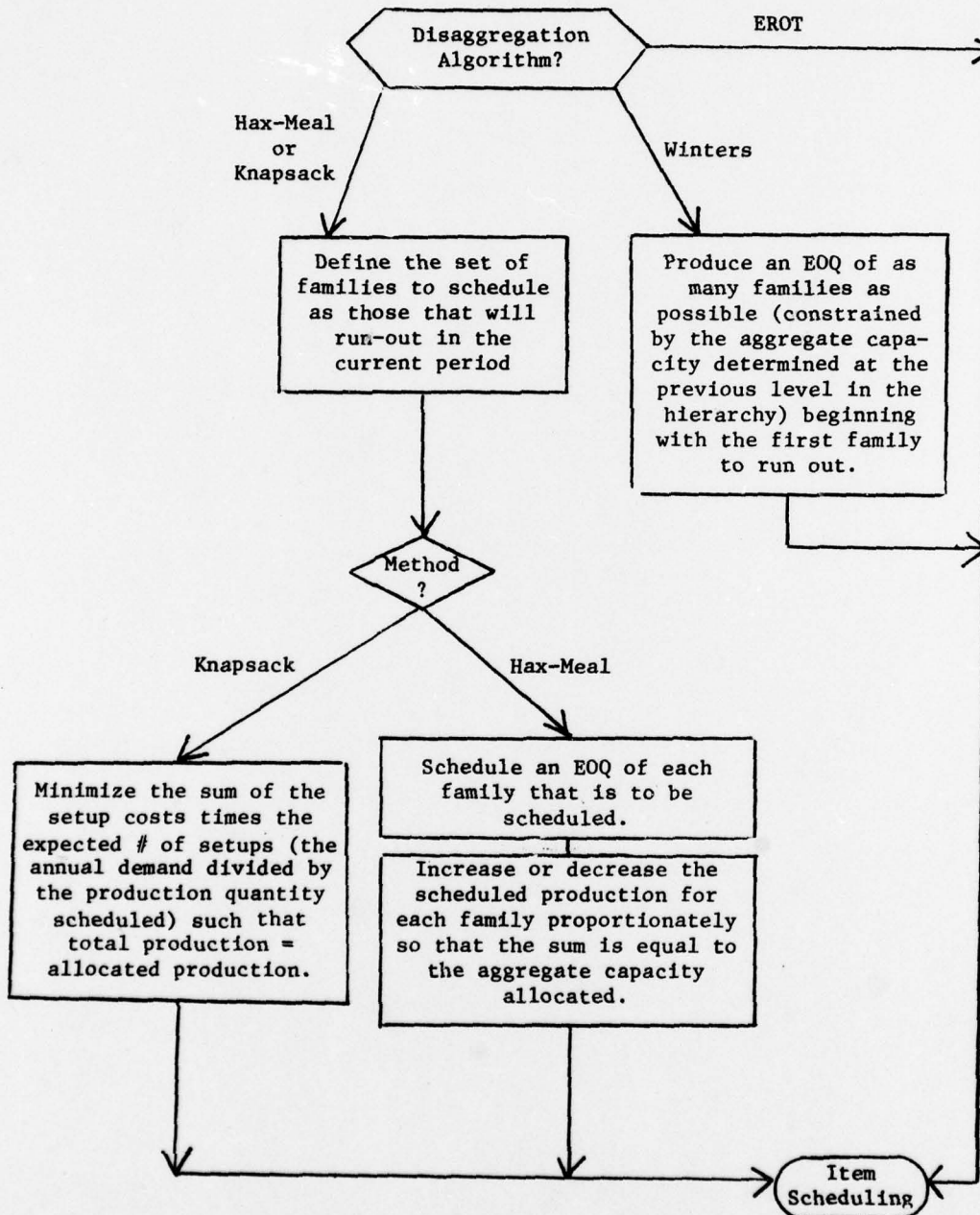


Figure 2: The Disaggregation Algorithms

The-Equalization-of-Run-Out-Times Approach (EROT)

EROT [12] defines run-out time to be the expected length of time until it is necessary to produce again (available inventory less the safety stock divided by the annual demand). The concept behind the EROT algorithm is to ignore setup costs for the current period and maximize the time until any item in the product type requires another setup. The primary concern is future costs. This approach disaggregates directly from product types to items.

The Winters Approach

The Winters approach was first proposed in 1962 [18]. Its central idea is to exploit the Economic Order Quantity (EOQ) principle. If the aggregate model sets production for a product type consisting of N families to be Y units, the Winters method schedules the production of an EOQ of as many of the N families as possible, while keeping the total production for the N families under Y. The families chosen to be produced in the current time period are selected in the order of their expected run-out times.

The Hax-Meal Approach

The Hax-Meal algorithm was developed in 1973 [11]. The concept behind this algorithm is to base initial allocations on the classical EOQ formula and then to adjust these levels to better fit the aggregate model. The Hax-Meal approach recognizes that holding costs over future periods have been determined by the aggregate model, and that the primary concern should be minimization of setup costs. This methodology allocates production capacity only to those families that will run-out in the immediate period in lot sizes proportional to their EOQ's. If overstock limits¹ are reached, other families are produced up to their overstock limits. For example, if the sum of the

¹Overstock limit is used for products that have limited sales seasons. It is aimed at maximizing expected return by balancing the cost of understock with that of overstock.

EOQ's of each of the products that will run out is less than the total production allocated, the Hax-Meal approach will increase the quantity of each family produced until total production is exactly equal to the capacity allocated, while the Winters approach will either produce an EOQ of another family, if there is sufficient capacity remaining, or else not use the remaining capacity.

The Knapsack Approach

The Knapsack was formalized in 1977 [1]. It was an attempt to build the Hax-Meal concepts into a sub-optimization model. This approach schedules the production of families to minimize expected setup costs over a year, in such a way that all allocated capacity is used.

The Item Model

The final stage in the hierarchical production planning process is the scheduling of items within a family (note: the EROT approach does this in the second stage). For this task, overstock limits and service requirements must be observed. The general approach to accomplishing this task is to equalize the expected run-out times for the items in a family. The expected time until it is necessary to set up the family again is thereby maximized. An alternate approach, similar to the Knapsack approach, has been proposed by Bitran and Hax [1], but is not being tested here.

THE STATISTICAL COMPARISON

When working with production planning, we believe there are two critical dimensions: costs and backorders. To compare the approaches, costs and backorders are generated over a full year. We felt that by using annual data we would avoid confusing random and seasonal differences with long run overall differences.

Previous comparisons have put a cost on backorders and reduced the comparative vector to one dimension. We believe that putting a cost on backorders would limit the effectiveness of the comparison. By separating backorders from costs we can impute the backorder costs which would make the methods comparable. In general, it is difficult to price the intangible concept of goodwill and backorders.

For our comparison, we generated a large data base, which was fairly representative of all situations that might use hierarchical production planning. The data used was representative of actual scenarios drawn from the authors' experiences. We held the product structure constant and varied the forecast error, available capacity, seasonality of demand for each product, family setup costs, and the planning horizon. The ranges of variation of these parameters were chosen to encompass all situations in which hierarchical production planning might be used. For a justification of the variables held constant and for a description of the ranges chosen, see Appendix 1.

The tests we used to compare the algorithms consisted of:

- 1) the signed Wilcoxon - on all data for overall pairwise comparisons of both costs and backorders,
- 2) the signed Wilcoxon - applied to quartile regions of each input variable,
- 3) the signed Wilcoxon - applied to prespecified regions defined by more than

one input variable,

- 4) a robustness comparison with respect to both costs and backorders, and
- 5) pointwise comparisons, for specific data points of interest.

In this section we present each of the tests and our interpretation of the results.

The Signed Wilcoxon Comparison

To identify distinguishing characteristics of the algorithms, we chose to compare the methods pairwise, using the Wilcoxon statistic [16]. The null hypothesis tested by this statistic is that the difference in the result of two methods applied to the same input conditions is randomly drawn from a continuous, symmetric distribution about zero. (For a fuller description of this statistic, see Appendix 2.)

We compared the output of 112 sets of input data, covering the entire input space tested. The output we compared consisted of total production costs (setup, holding, regular-time and overtime costs summed over thirteen periods of 4 weeks each), and total backorders.

The results of the Wilcoxon tests are displayed in Figure 3. From this broad test, two conclusions can be tentatively drawn.

- 1) The Winters algorithm is the least desirable. (Note: this conclusion is strengthened by later tests.)
- 2) The Knapsack method compares favorably with respect to costs and unfavorably with respect to backorders. In the aggregate, for the Knapsack method to be equivalent to the Hax-Meal approach, backorders would have to be valued at \$1.07/unit/period. For the Knapsack method to be equivalent to the EROT approach, backorders would have to be valued at \$1.89/unit/period. The holding costs used in the modeling effort were set at \$.38/unit/period and may be viewed as a minimum backorder cost.

		Methods being compared via the Wilcoxon Test					
Decision Criteria		Hax-Meal Winters	Hax-Meal EROT	Winters EROT	Hax-Meal Knapsack	Knapsack EROT	Knapsack Winters
		HM	-	EROT	KN	KN	KN
Total Costs		HM	-	EROT	KN	KN	KN
Backorders		HM	-	EROT	HM	EROT	KN

HM = Hax-Meal
 WI = Winters
 KN = Knapsack

Letters in the boxes indicate dominance by more than 2 standard deviations.
 - indicates no clear dominance.

Figure 3: The Wilcoxon Applied to all Data

Sectional Wilcoxon Tests

We performed the sectional tests in two parts. We first broke each input variable into quartiles and tested the data in each quartile for each input variable. Then we identified specific regions of interest, defined by more than one input variable (i.e. high forecast error and high seasonality), and tested the data in each of these regions.

The quartile tests were done to narrow down the ranges of dominance. For example, as forecast error increased so did the Hax-Meal dominance over the Winters approach with respect to backorders (see Figure 4). For a summary of all the results, see Figure 5.

From the data presented in Figure 5, the following conclusions were drawn:

- 1) there is no general situation in which the Winters approach dominates, with respect to costs or backorders;
- 2) when seasonality is high or setup costs are high, EROT does very poorly with respect to costs. It is the only method ever dominated by the Winters approach;
- 3) when setup costs are in their lowest quartile, EROT dominates both the Hax-Meal approach and the Winters approach; and
- 4) when capacity is at its tightest, Hax-Meal dominates all other methods.

When examining regions defined by multiple criteria, we divided each input variable into two ranges. For each variable, the "high" and "low" ranges were defined to be above or below the median, respectively. The output of the Wilcoxon test applied to these prespecified regions, presented in Figure 6, indicate that:

(Grouped by forecast error into columns - and then ordered within columns by the magnitude of the difference.)

<u>0-7.5%</u>	<u>7.5-15%</u>	<u>15-22.5%</u>	<u>22.5-30%</u>
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	3	7
0	0	4	-163
0	0	-19	-387
0	0	-134	-424
0	0	-151	-517
0	0	-389	-533
0	0	-453	-743
0	0	-480	-962
0	0	-521	-1021
0	0	-611	-1219
0	0	-662	-1295
0	-1	-690	-1328
0	55	-755	-1518
0	138	-757	-1570
-5	-298	-914	-1790
42	-305	-1023	-1906
-74	-547	-1171	-2430
-164	-696	-1414	-3786
-263	-1754	-1457	-5267
243	-2147	-1649	-6095
-2399	-5188	-2252	-7214
-6267	-5249	-4056	-10586
-8772	-7000	-25083	-10900

Statistics associated with the above numbers

	<u>the first quartile</u>	<u>the second quartile</u>	<u>the third quartile</u>	<u>the fourth quartile</u>
Mean difference	-1740	-1916	-1941	-2680
# of non-zero values	9	12	23	23
Wilcoxon	-29	-68	-270	-274
Std. dev. of Wilc.	16.9	25.5	65.8	65.8

Figure 4: Hax-Meal Backorders Less Winters Backorders

Decision Criterion	Hax-Meal vs. Winters		Hax-Meal vs. EROT		EROT vs. Winters		Hax-Meal vs. Knapsack		Winters vs. Knapsack		EROT vs. Knapsack	
Overall Cost	HM 1.6		-		EROT 2.13		KN 2.3		KN 1.9		KN 1.7	
Costs Ordered by:												
Backorders of Product 2	-	HM 2.1	-	EROT 2.3	-	EROT 2.3	-	-	KN 2.6	-	-	KN 2.1
	HM 2.5	HM 1.8	EROT 1.8	-	EROT 3.2	WI 2.1	-	-	-	WI 1.7	-	-
Forecast error	HM 1.7	HM 2.7	-	-	-	EROT 1.8	KN 1.8	-	KN 1.7	-	-	-
	HM 4.1	HM 4.2	-	-	EROT 2.0	-	-	-	-	-	-	-
Capacity	HM 4.2	-	HM 3.6	-	EROT 4.1	-	HM 1.7	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	KN 1.8	-	EROT 1.6
Setup Costs	-	-	EROT 2.5	-	EROT 1.9	-	-	-	-	-	EROT 1.5	-
	HM 2.3	HM 2.8	HM 2.4	HM 1.4	-	WI 1.8	-	KN 2.39	-	-	-	KN 2.8
Overall Seasonality	HM 2.2	HM 2.1	EROT 3.2	EROT 2.9	EROT 2.7	EROT 2.1	KN 2.1	-	KN 2.3	-	-	-
	-	-	-	HM 1.7	-	WI 1.7	HM 1.6	HM 1.8	-	-	-	-
Planning Horizon	-	HM 2.5	EROT 1.8	-	-	EROT 2.2	KN 2.3	-	-	KN 2.0	KN 1.9	EROT 1.6
Overall Backorders	HM 6.58		-		EROT 6.58		HM 2.2		KN 1.6		EROT 1.6	
Backorders Ordered by:												
Forecast error	HM 1.7	HM 2.7	-	-	-	EROT 3.1	-	-	-	-	-	-
	HM 4.1	HM 4.2	-	HM 2.8	EROT 3.7	EROT 4.1	HM 2.1	HM 2.2	KN 2.7	KN 2.5	-	EROT 1.5
Seasonality of Product Type 2	HM 3.6	HM 3.6	-	-	EROT 3.1	EROT 3.9	-	-	-	-	-	-
	HM 3.4	HM 3.1	-	-	EROT 3.3	EROT 2.0	HM 2.0	HM 2.3	-	-	-	EROT 1.7
Capacity	HM 3.8	HM 3.2	-	-	EROT 3.8	EROT 3.0	HM 1.9	HM 1.7	KN 2.5	KN 2.2	-	EROT 2.0
	HM 3.5	HM 2.8	-	-	EROT 3.6	EROT 2.9	-	-	KN 1.8	-	-	-
Setup Costs	HM 3.2	HM 3.7	-	-	EROT 2.9	EROT 4.0	HM 1.7	HM 2.0	-	-	EROT 1.6	-
	HM 3.2	HM 3.1	-	HM 1.9	EROT 3.2	EROT 3.2	-	-	KN 2.4	KN 2.0	-	-
Planning Horizon	HM 4.69	-	-	-	EROT 4.9	-	HM 1.6	HM 2.01	-	-	EROT 1.5	EROT 2.0
	HM 4.56	-	-	-	EROT 4.4	-	-	-	-	-	-	-

Notes: The numbers in the boxes indicate how many standard deviations away from zero the results are. Only statistically significant results are shown. When ordering occurs the boxes are divided as follows:

1st quartile	2nd quartile
3rd quartile	4th quartile

Figure 5: A Detailed Comparison of the Algorithms

Regional Definition \ Methods being compared	Hax-Meal	Hax-Meal	Hax-Meal	Winters	Winters	EROT
	Winters	EROT	Knapsack	EROT	Knapsack	Knapsack
high setup costs low capacity	HM	HM	KN	WI	-	KN
	HM	-	-	EROT	KN	-
low setup costs high forecast error	-	EROT	-	EROT	-	-
	HM	EROT	HM	EROT	KN	EROT
high forecast error low capacity	HM	HM	-	EROT	-	-
	HM	-	-	-	KN	-
short planning (6 mo. horizon) high seasonality	-	-	-	EROT	-	KN
	HM	-	HM	-	-	EROT
high forecast error high seasonality	HM	-	-	-	-	-
	HM	-	HM	EROT	-	-

Notes: Dominance means that the Wilcoxon is more than 2 standard deviations from zero. The initials in the boxes indicate:

cost dominance
Backorder dominance

Figure 6: Comparisons of Well-Defined Regions

- 1) We could not identify a situation where we would recommend the use of the Winters approach (reconfirming the indication given by the first test);
- 2) EROT performs most poorly in situations with a combination of high setup costs and low forecast error; and
- 3) EROT outperforms all other approaches tested when forecast error is high and setup costs are low.

The Robustness Test

The robustness of each method was measured by examining its deviations from an "ideal" vector constructed by combining the lowest backorders and costs achieved by any method. For each method, a delta vector was defined to be the difference between the actual results and the "ideal" results. From this delta vector we extracted the eleven largest elements, 10% of the total data, and averaged them. Essentially, we measured how far the worst points generated by each method deviated from the "ideal" costs and "ideal" backorders. The results are given in Figure 7.

The following conclusions were drawn from this test:

- 1) the Hax-Meal method is the most robust;
- 2) the Winters method is the least robust (the backorder measure is 31 times as great as the Hax-Meal approach).

The data indicated that when products had high setup costs the Winters approach had its maximum deviations with respect to backorders. To further test the second conclusion above we simulated additional scenarios with setup costs higher than we had previously allowed. (We let setup costs range

Measure of "Robustness"			
	Costs	Backorders	Backorder cost to equate method with Winters
Hax-Meal	8.3K	307	\$.08/unit/period
Winters	7.6K	9304	
EROT	11.9K	1105	\$.52/unit/period
Knapsack	10.6K	618	\$.35/unit/period

mean 10% delta

Figure 7: A Robustness Comparison

Note: The holding cost used was \$.38/unit/period and is an absolute minimum backorder cost. It is the cost one would use if no cost was associated with loss of goodwill.

from 2300 to 3300.) The five points generated reconfirmed this conclusion. On all of these points the Winters method had the greatest number of backorders, at least three times that of Hax-Meal.

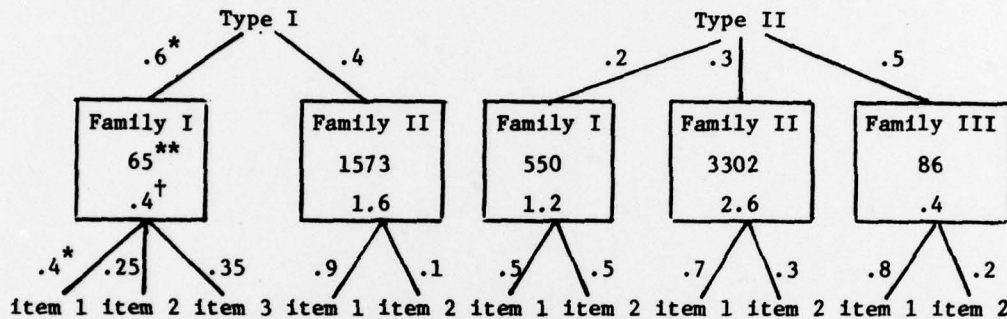
Pointwise Comparisons

The EROT approach performed differently for different levels of setup costs (see Figure 5). We thought that the EROT approach would not perform well when setup costs of different families within a product type were varied. We ran a limited set of data testing this hypothesis (6 points across all methods). The hypothesis appears to be valid. EROT had the highest costs on all six points, and never had the lowest backorders. See Figure 8 for an example.

THE INPUT DATA

Forecast error = 25%
 Demand of Product type I = 100,000 units
 Demand of Product type II = 120,000 units
 Seasonality = High + Period. Demand for product type II varies between 2639 and 15593.
 Capacity = Tight

Product Structure and Setup Costs



- * row of numbers is the share of total demand allocated to each family or items
 ** row of numbers is the setup cost
 † row of numbers is the number of periods demand 1 EOQ.

THE OUTPUT DATA

Method	Total Cost	Setup Cost	Backorders
Hax-Meal	169150	69186	7307
Winters	167816	65884	10449
EROT	172329	72488	7307
Knapsack	165412	65334	7867

- Typical** → 1. Winters has a large number of backorders.
 2. EROT has the largest total setup cost.
 3. When seasonality and forecast error are high, the Knapsack approach has a large amount of backorders.

Note: For Hax-Meal to be equivalent to Knapsack, backorders would need to be priced at \$6.67 per unit period.

Figure 8: An Example Point with varying setup costs

CONCLUSIONS

The Winters Approach

The strongest conclusion that can be drawn from our work is that the Winters approach is the poorest, regardless of the dimension being measured - costs, backorders, or robustness.

Intuitive reasoning identifies two factors that contribute to its poor performance. First, if the entire list of families in a type is to be produced, the concept of multiple EOQ's for some families is meaningless. The EOQ was designed to balance setup costs and holding costs. If more than one EOQ of some good is produced, more than one setup cost is not incurred. Offsetting a second setup cost against holding costs is meaningless. Second, the Winters approach usually does not produce up to the capacity allocated in the aggregate model. Typically an EOQ of some number of families is produced, with the sum of the EOQ's less than the allocated capacity, but the difference is not large enough to allow an EOQ of another product to be produced. As setup costs increase, so do EOQ's and the difference between allocated capacity and used capacity. When capacity is tight this under-producing can lead to excessive backorders. It is also possible for the allocated capacity to be less than that required to produce an EOQ of all products that will run out in the immediate period. All other approaches guarantee production of all items that will run out in the immediate period, while the Winters approach does not.

The Winters approach resembles the approach that is most extensively used in practice - the order-point order-quantity approach with the order-quantity being represented by an EOQ and the order-point being represented by the run-out time. Our results indicate that substantial

production planning improvements are obtainable for companies using an order-point order-quantity system by simply switching to a more robust hierarchical production planning methodology.

The EROT Approach

Two distinct conclusions have been supported by the data regarding the EROT approach.

- 1) The EROT approach should be used if
 - a) setup costs are low (an EOQ is $\leq 3/4$ period),
 - b) forecast error is high (greater than 24%), and
 - c) all families have identical cost structures in each product type.
- 2) The EROT approach should be avoided if either
 - a) setup costs vary between families, or
 - b) setup costs are high (so that an EOQ > 1 period's demand).

These results are also intuitively justifiable. If the sole objective was to minimize backorders, constrained by the aggregate plans, the EROT approach would be ideal. The probability of any family running out is minimized with the EROT approach, as inventory is distributed evenly over all items¹. However, if there is a small forecast error nothing may be gained by the EROT approach. Our data indicates that forecast error must be greater than 24% for this difference to be noticed. Hax-Meal outperformed the EROT with forecast error between 18 and 23% by more than EROT outperformed Hax-Meal with forecast error between 24 and 30%.

Differing setup costs impair the effectiveness of the EROT approach. The EROT approach does not discriminate based on setup costs and typically produces all families in every period. When one family's setup cost is high relative to another family's, more of the family with the high setup cost should be produced in order to minimize future costs. The

¹This statement assumes that the forecast errors of all families in a product type have identical distributions.

purpose of this is to eliminate the need to setup for the higher cost product in the next period.

The Knapsack Approach

The Knapsack approach is the best approach with respect to costs, and is relatively robust, second only to the Hax-Meal approach. It is the only method that 'optimizes' with respect to setup costs while disaggregating. When setup costs are high, forecast errors are low and seasonality is low the Knapsack approach dominates.

The algorithm implicitly assumes no forecast error and no seasonality. When these assumptions are far from the truth, the Hax-Meal approach outperforms the Knapsack approach. This is explainable by the form of the model. The disaggregation model minimizes, on an annual basis, the number of setups necessary for each family. A more myopic view may work better when seasonality is pronounced. For example, the annual demand figure may indicate that setups will occur once every other month. When demand is at its peak, more setups may be necessary and when it is at its trough a setup every other month may still be necessary if the production size is poorly set.

The Hax-Meal Approach

The Hax-Meal algorithm has performed very well throughout the testing procedure. There were only two very specific cases in which another approach could be identified as outperforming the Hax-Meal algorithm (see Figure 9). The principal strength of the Hax-Meal approach is that it is phenomenally robust, and produces good results over a wide range of production planning situations. Overall, it is the method we would recommend.

CONDITION	SUGGESTED METHOD BASED ON OUR TESTS
Low setup costs ($EOQ \leq 3/4$ period) and identical cost structures for all products in a type, high forecast error.	EROT
Low Seasonalities, Loose Capacity, High setup Costs, Low Forecast Error	Knapsack
All other situations	Hax-Méal

Figure 9: Summary of Conclusions

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APPENDIX 1: THE DATA BASE

Given the large number of possible input parameters, we varied those we felt were most important to test and held constant the holding cost, minimum service level, annual demand, productivity, and the product structure. All variables held constant were set to values representative of an actual large manufacturer of tires.

The minimum service level was set at 95%. Varying the minimum service level would only alter the safety stock. There would be no change in the relative merits of one method versus another. If the service level were increased, all methods would have a greater holding cost and all backorders would decrease by a fixed number.

We believe that the productivity would principally affect the aggregate model and not change the relative strengths of any of the disaggregation approaches.

The product structure we chose to use throughout the simulation is illustrated in Figure A1. Items sharing molds in the tire curing process, and therefore sharing setup costs, were grouped into families. Families with similar seasonal patterns were grouped into types. The annual demand was set to 100,000 units of product type I and 120,000 units of product type II.

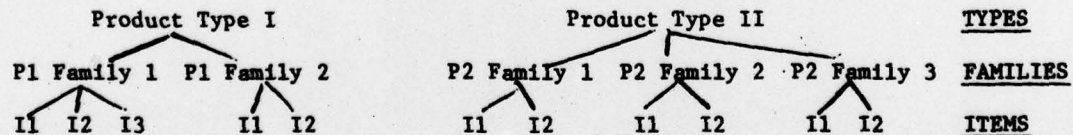


Figure A1: Product Structure

We did not feel it was necessary to vary both the holding costs and the setup costs, as altering either one alters the economic order quantity and it is their relative magnitudes that are important for the purpose of

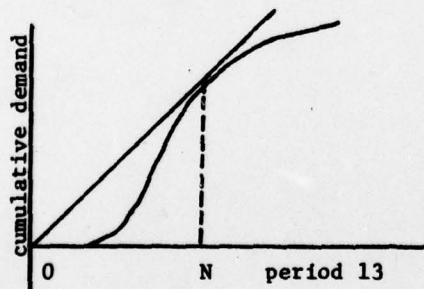
simulation, We chose to vary setup costs, and fixed holding costs at 25% of total unit costs.

The economic order quantity run lengths were varied between one quarter of a period and just over 2 periods, by altering the setup costs. It was felt that economic order quantities larger than 2 periods would no longer have setup costs secondary in nature, a fundamental assumption of the hierarchical approach to production planning. Setting the economic order quantity between 1/4 and 2 periods resulted in setup costs varying from .3 to just under 15 percent of the total production costs.

Capacity was varied between 0 and 100, and was measured in the following manner.:

- (a) Determine at which of the thirteen periods there exists the highest average demand per period (based on cumulative demand - see Figure A2).

Let us call this period N.



Note: the point where the tangent from the origin intersects the cumulative demand curve is the point of highest average demand per period.

Figure A2

- (b) At period N compute the average demand per period, or:

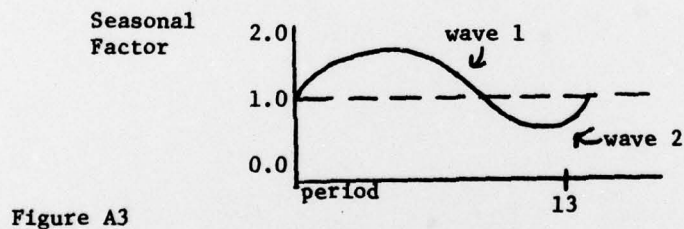
$$\text{Avg. Demand}_N = \text{Cumulative demand up to period N} / N.$$

- (c) If capacity with no overtime is equal to avg. demand_N, then capacity is set to 100. If capacity with the maximum allowable overtime is equal to avg. Demand_N, then capacity is set to 0. At all points

between these two extremes, capacity is scaled appropriately.

Limits on forecast errors were fixed at a given percentage of demand. Actual forecast errors were randomly distributed in the allowable range. The limits on forecast error varied between zero and thirty percent of demand.

Each demand pattern was treated as 2 connected sine waves (see Figure A3), with peaks and valleys constrained to be less than one unit away from the center. The waves could meet at any of the 13 periods. It was required that the waves' directions be opposite to one another. The seasonal factors were then normalized, so that the average was equal to 1.0. As a measure of seasonality, the coefficient of variation was calculated for each demand pattern.



We chose to examine 2 distinct planning horizons, 6 periods and 13 periods, as these are the timings which are typically used for production planning. (We defined a period to be equal to four weeks.)

APPENDIX 2: THE WILCOXON TEST

The Wilcoxon statistic might be thought of as a cross between the t-statistic, which makes assumptions about the distribution of the data and is a function solely of the magnitude of the differences, and the sign test, which totally ignores the size of the differences. The Wilcoxon test was originally designed for tests performed on pairs of twins. The data which we generated fits well into this type of comparison, as pairwise points were points generated from identical situations via different algorithms. The use of matched pairs produces a powerful statistic, as it reduces the variance of the differences between points and thereby enables us to obtain a more accurate statistic with tighter confidence limits.

The Wilcoxon statistic is computed in the following manner:

- 1) Pair the data to be compared.
- 2) Compute the difference in all pairs of data.
- 3) Rank the differences by their absolute value.
- 4) To the rank of the i^{th} absolute difference, attach the sign of the difference. Denote this signed rank by r_i .
- 5) The Wilcoxon statistic is equal to the sum of the signed ranks,

$$W = r_1 + r_2 + \dots$$

For a more in depth description of this statistic, the reader is referred to Mosteller [15].